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Stress Analysis of Functionally Graded Disc under Thermal and Mechanical Loads

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Abstract

In this study, the elastic stress analysis of annular discs made of functionally graded materials (FGMs) subjected to both uniform pressures on the inner surface and a linearly decreasing temperature distribution has been carried out. The elasticity modulus and thermal expansion coefficient of the discs are assumed to vary radially according to power law functions and Poisson's ratio is kept constant. Analytical solutions for the elastic stress in annular discs are obtained under plane stress assumption. Calculation programs were prepared by authors using MATLAB 7.1. The change of the stresses, displacements, elasticity modulus and thermal expansion coefficient according to the gradient parameters were investigated and presented.

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Keywords: Functionally graded materials; Disc; Thermal load; Thermal stress.

1. Introduction

Functionally graded materials (FGMs) are a new kind of heterogeneous composite material that consists of a gradient compositional variation from one surface to another. This compositional variation in properties improves the thermal and mechanical behaviors of the system.

In literature, there are various studies on thermal and mechanical stress analyses of functionally graded materials [1-3]. The stress distribution in a nonhomogeneous anisotropic cylindrical body is investigated

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by Oral and Anlas [4]. Chareonsuk and Vessakosol [5] presented numerical solutions for functionally graded solids under thermal and mechanical loads using a high-order control volume finite element method. A significant amount of work has been done in order to understand thermal response and thermodeformations in FGM that subjected to thermal and mechanical loads and optimize the physical and thermomechanical properties of FGMs.

The aim of this study is to obtain the solution for elastic stress analysis of annular discs made of FGMs subjected to thermal and mechanical loads by an analytical method. Analytical solutions provide invaluable checks on the accuracy of numerical or approximate analytical schemes and allow widely applicable parametric studies.

2. Thermal Elastic Stress Solution

A FGM annular disc subjected to both uniform pressures on the inner surface and a linearly decreasing temperature distribution. Linearly-decreasing temperature equation can be written as

$$\frac{T}{T_0} = \frac{r_o - r}{r_o - r_i}, \qquad T = T_0 \frac{r_o - r}{r_o - r_i} \tag{1}$$

in which r_i and r_o are inner and outer radius of FG disc, respectively. The temperature in the outer surface is zero and in the internal surface, T_0. The elasticity modulus and thermal expansion coefficient of the discs are assumed to vary radially according to two diverse power law functions. It is assumed that

$$E(r) = E_0 \left(\frac{r}{r_i}\right)^n, \qquad \alpha(r) = \alpha_0 \left(\frac{r}{r_i}\right)^m \qquad \text{(for disc 1)} \tag{2.a}$$

$$E(r) = E_0 \left(\frac{r}{r_o}\right)^n, \qquad \alpha(r) = \alpha_0 \left(\frac{r}{r_o}\right)^m \qquad \text{(for disc 2)} \tag{2.b}$$

where n and m are dimensionless arbitrary constants (gradient parameter) of elasticity modulus and thermal expansion coefficient, respectively. When *n* and *m* are equal to zero, the disc becomes isotropic. The governing differential equation of equilibrium for the plane stress can be written as

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{3}$$

Because of the symmetry, the value of shear stress, $\tau_{r\theta}$, is equal to zero. Stress components are independent of θ. For elastic materials, the relationship between the strains and stresses that take place on an FG disc under the influence of temperature can be described by Hooke's law

$$\varepsilon_r = \frac{1}{E(r)}(\sigma_r - \nu\sigma_\theta) + \alpha(r)T, \quad \varepsilon_\theta = \frac{1}{E(r)}(\sigma_\theta - \nu\sigma_r) + \alpha(r)T \tag{4}$$

where $\alpha(r)$ and $E(r)$ are varying thermal expansion coefficient and elasticity modulus, respectively. In terms of the Airy stress function, F, stresses σ_r and σ_θ are given by

$$\sigma_r = \frac{F}{r}, \quad \sigma_\theta = \frac{dF}{dr} \tag{5}$$

Strain-displacement relation can be written as

$$\varepsilon_r = \frac{du}{dr}, \quad \varepsilon_\theta = \frac{u}{r} \tag{6}$$

in which u is the displacement component in the radial direction. The deformation compatibility equation can be derived from Eq.(6) which have the following form.

$$\varepsilon_r = \frac{d}{dr}\left(r\varepsilon_\theta\right) \tag{7}$$

From this Eq. (6), it is seen that the two strain components are related by Eq. (7). By making use of Eq. (4) into this relation, one can readily obtain

$$r^2\frac{d^2F}{dr^2} + r(1-n)\frac{dF}{dr} + (n\nu - 1)F = -mE(r)\alpha(r)rT - E(r)\alpha(r)r^2\frac{dT}{dr} \tag{8}$$

Eq. (8) is the governing second order differential equation. Once this equation is solved for F, the components of stress, strain, and displacement can readily be found. Substituting equations Eq. (1) and Eq. (2) into the equation (8) and the stress function, F, can be written as

$$F = C_1 r^{\frac{n+k}{2}} + C_2 r^{\frac{n-k}{2}} + Gr + Hr^2 \tag{9}$$

where C_1 and C_2 are the integration constants and the positive constant k is

$$k = \sqrt{n^2 - 4n\nu + 4} \tag{10}$$

and the term G and H are;

$$G = \frac{-mE(r)\alpha(r)r_iT_0}{(r_o - r_i)(n\nu - n)}, \qquad H = \frac{(m+1)E(r)\alpha(r)T_0}{(r_o - r_i)(3 + n(\nu - 2))} \tag{11}$$

σ_r and σ_θ stress components can be obtained from the stress function as,

$$\sigma_r = C_1 r^{\frac{n+k-2}{2}} + C_2 r^{\frac{n-k-2}{2}} + G + Hr, \quad \sigma_\theta = \left(\frac{n+k}{2}\right)C_1 r^{\frac{n+k-2}{2}} + \left(\frac{n-k}{2}\right)C_2 r^{\frac{n-k-2}{2}} + G + 2Hr \tag{12}$$

The integration constants, C_1 and C_2 can be obtained from the boundary conditions. FG disc subjected to uniform pressures on the inner surface and no radial displacement of the outer surface.

$\sigma_r = -P$ at the $r = r_i$ and u=0 at the $r = r_o$

By using these conditions, C_1 and C_2 are determined as

$$C_1 = \frac{-(G + Hr_i + P)\dfrac{r_o^{\frac{n-k}{2}}}{E(r_o)}\left(\dfrac{n-k}{2} - \nu\right) + \left(\dfrac{Gr_o}{E(r_o)}(\nu - 1) + \dfrac{Hr_o^2}{E(r_o)}(\nu - 2) - \alpha(r_o)r_oT(r_o)\right)r_i^{\frac{n-k-2}{2}}}{r_i^{\frac{n+k-2}{2}}\dfrac{r_o^{\frac{n-k}{2}}}{E(r_o)}\left(\dfrac{n-k}{2} - \nu\right) - r_i^{\frac{n-k-2}{2}}\dfrac{r_o^{\frac{n+k}{2}}}{E(r_o)}\left(\dfrac{n+k}{2} - \nu\right)} \tag{13.a}$$

$$C_2 = \frac{\left(\frac{Gr_o}{E(r_o)}(\nu-1) + \frac{Hr_o^2}{E(r_o)}(\nu-2) - \alpha(r_o)r_oT(r_o)\right)r_i^{\frac{n+k-2}{2}} + (G+Hr_i+P)\frac{r_o^{\frac{n+k}{2}}}{E(r_o)}\left(\frac{n+k}{2}-\nu\right)}{r_i^{\frac{n+k-2}{2}}\frac{r_o^{\frac{n-k}{2}}}{E(r_o)}\left(\frac{n-k}{2}-\nu\right) - r_i^{\frac{n-k-2}{2}}\frac{r_o^{\frac{n+k}{2}}}{E(r_o)}\left(\frac{n+k}{2}-\nu\right)} \qquad (13.b)$$

Radial displacement, u, in the elastic solution for small deformation can be determined from equation (9) as,

$$u = \frac{1}{E(r)}\left[\left(\frac{n+k}{2}\right)C_1 r^{\frac{n+k}{2}} + \left(\frac{n-k}{2}\right)C_2 r^{\frac{n-k}{2}} + Gr + 2Hr^2 - \nu\left(C_1 r^{\frac{n+k}{2}} + C_2 r^{\frac{n-k}{2}} + Gr + Hr^2\right)\right] + \alpha(r)rT \qquad (14)$$

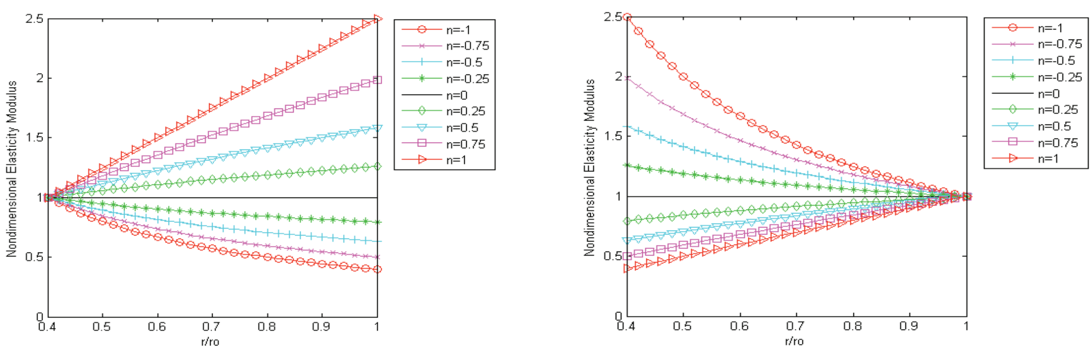
radial displacement, circumferential and radial stress components under both the load of linearly-decreasing temperature and uniform pressures on the inner surface can be found in this way.

3. Results and discussion

Stresses, displacement, elasticity modulus and thermal expansion coefficient along the radial direction of the discs are given as dimensionless values to demonstrate the effects of FGMs on the discs. For the presentation of numerical results, following formal dimensionless variables are used:

$$\bar{E} = \frac{E(r)}{E_0},\ \bar{\alpha} = \frac{\alpha(r)}{\alpha_0},\ \bar{\sigma}_r = \frac{\sigma_r}{E_0\alpha_0T_0},\ \bar{\sigma}_\theta = \frac{\sigma_\theta}{E_0\alpha_0T_0} \text{ and } \bar{u} = \frac{u}{r_o\alpha_0T_0} \qquad (15)$$

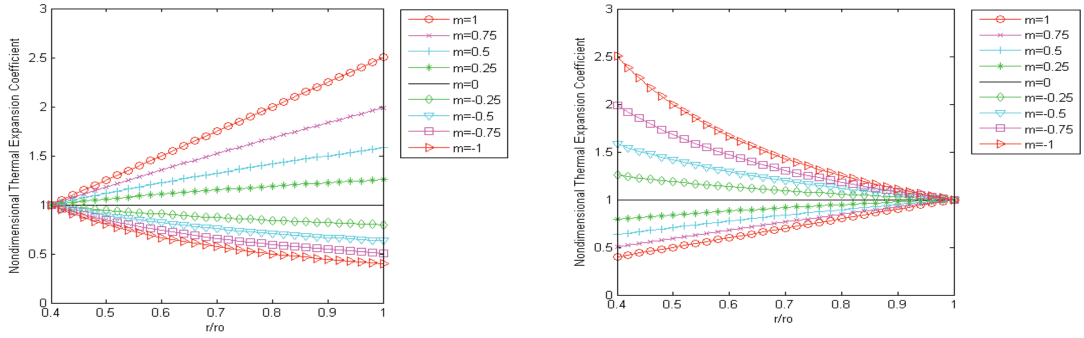
Figures 2 (a) and (b) illustrate dimensionless elasticity modulus variations, $\bar{E}$, for discs 1 and 2, respectively. As seen in these figures, a positive n means increasing $\bar{E}$ while a negative n means decreasing $\bar{E}$ along the radial direction for discs 1 and 2.



(a) (b)

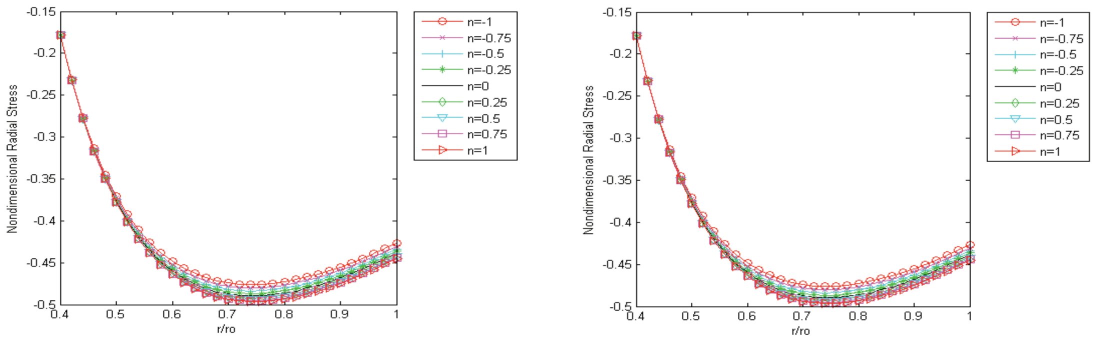
Figure 2. Variations of the dimensionless elasticity modulus $\bar{E}$, (a) for disc 1, (b) for disc 2.

Figures 3 (a) and (b) show variation of the dimensionless thermal expansion coefficient, $\bar{\alpha}$, for discs 1 and 2, respectively. As seen in these figures, a negative m means decreasing $\bar{\alpha}$ while a positive m means increasing $\bar{\alpha}$ along the radial direction for discs 1 and 2.



(a) (b)

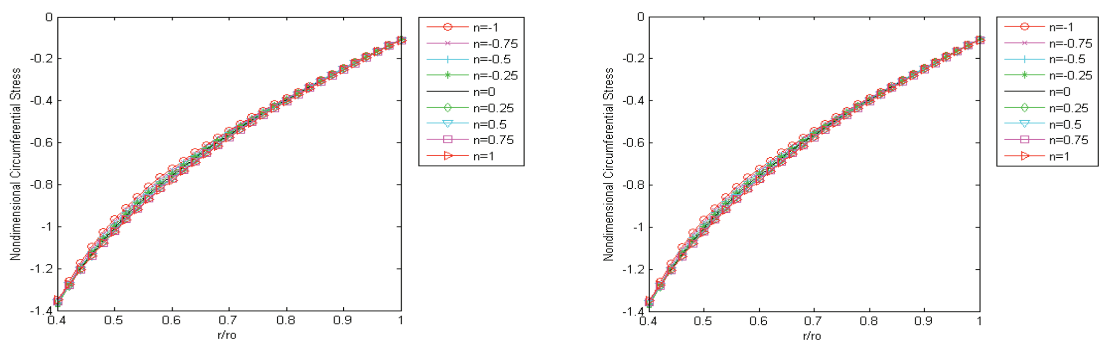
Figure 3. Variations of the dimensionless thermal expansion coefficient $\bar{\alpha}$, (a) for disc 1, (b) for disc 2.



(a) (b)

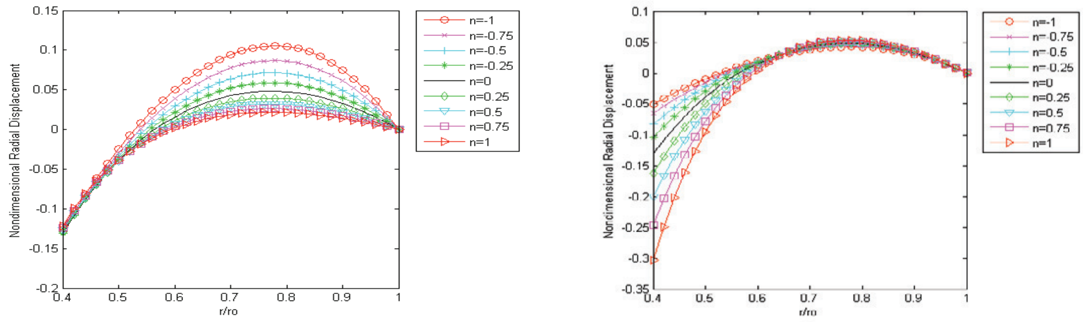
Figure 4. Variations of the dimensionless radial stress $\bar{\sigma}_r$, (a) for disc 1, (b) for disc 2.

Variation of the dimensionless radial stress, $\bar{\sigma}_r$, is depicted in Figures 4 (a) and (b), for disc 1 and 2, respectively. As seen in these figures, radial stresses in both discs 1 and disc 2 has the maximum value for *n=+1*, and with decreasing *n* values it decreases gradually and continuously in all the discs. Distribution of the dimensionless circumferential stress, $\bar{\sigma}_\theta$, along the radial direction is illustrated in Figures 5 (a) and (b), for disc 1 and 2, respectively.



(a) (b)

Figure 5. Variations of the dimensionless circumferential stress $\bar{\sigma}_\theta$, (a) for disc 1, (b) for disc 2.



(a) (b)

Figure 6. Variations of the dimensionless displacement $\bar{u}$, (a) for disc 1, (b) for disc 2.

Figures 6 (a) and (b) demonstrate variation of the dimensionless displacement, $\bar{u}$, for disc 1 and 2, respectively.

4. Conclusions

The change of the stresses, displacements, elasticity modulus and thermal expansion coefficient according to the gradient parameters were investigated and the results can be summarized as follows:

- Disc 1 has the highest elasticity modulus value, $\bar{E}$, at the outer edge for n=+1, while disc 2 has at the inner edge when n=-1
- Thermal expansion coefficient, $\bar{\alpha}$, disc 1 has the highest value at the outer edge, for $m=+1$, while disc 2 has the highest value at the inner edge when $m=-1$.
- The magnitudes of circumferential stress components are bigger than radial stress components at the inner surface, but lower than at the outer surface.
- Circumferential stress has the highest value at the inner surface whereas it is the lowest value at the outer surface in all the discs.
- Radial stress is the highest in both discs 1 and disc 2, for $n=+1$, and with decreasing n values, it decreases gradually and continuously in all the discs.
- Displacement, $\bar{u}$, disc 1 has the highest value at middle plane for $n=-1$, while disc 2 has the highest value for $n=+1$ at the inner edge.

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